Spin-Orbit Coupling in an Unpolarized Heavy Nucleus

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Yuri Kovchegov and M.S., Phys.Rev. **D89** (2014) 5, 054035

and a paper in preparation

Outline

I. How Do You Define a Quark Distribution?

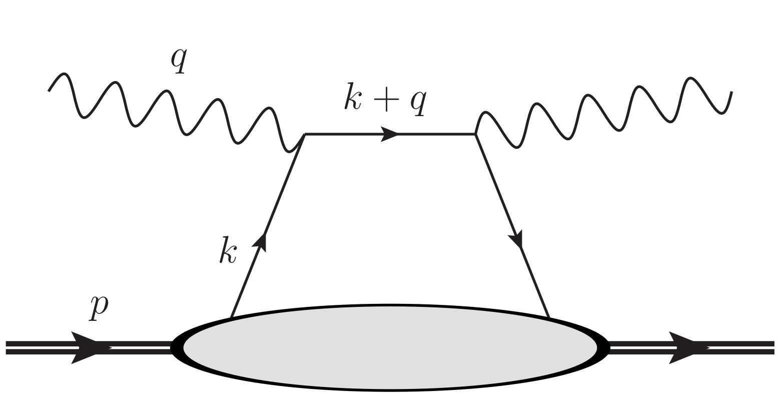
- Collinear PDF's
- Transverse-Momentum Dependent PDF's

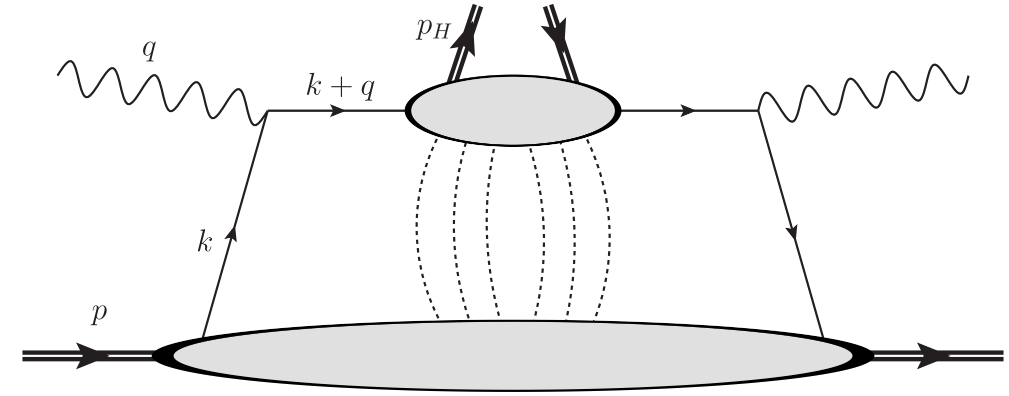
II. The Power of the High-Density Limit

- The McLerran-Venugopalan Model of a Heavy Nucleus
- Quasi-Classical Factorization of the TMD's

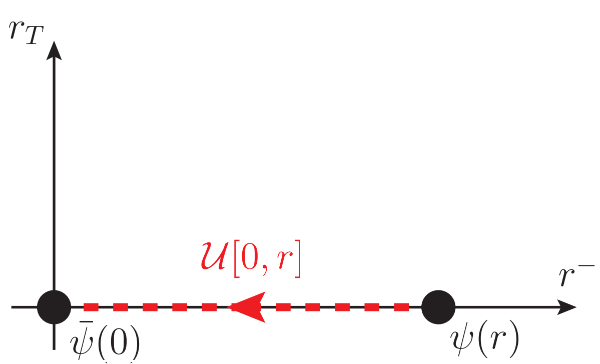
III. Spin-Orbit Coupling in an Unpolarized Nucleus

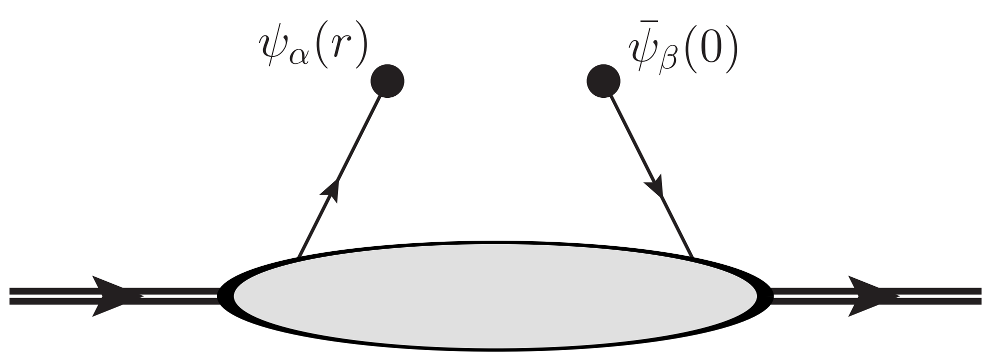
- Spin-Orbit Structure of the Nucleus
- Implications for the Nuclear TMD's

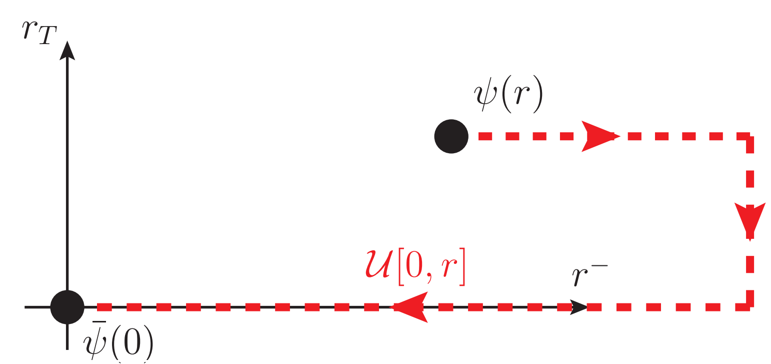


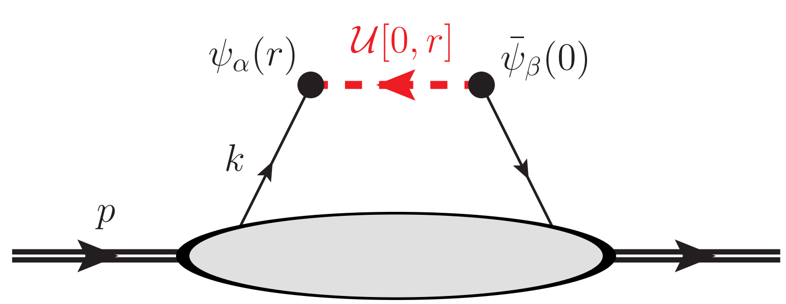


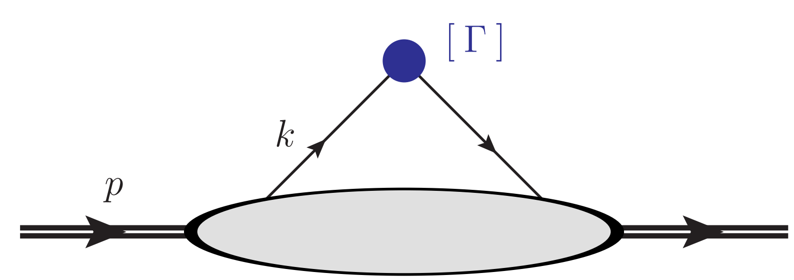
How Do You Define a Quark Distribution?









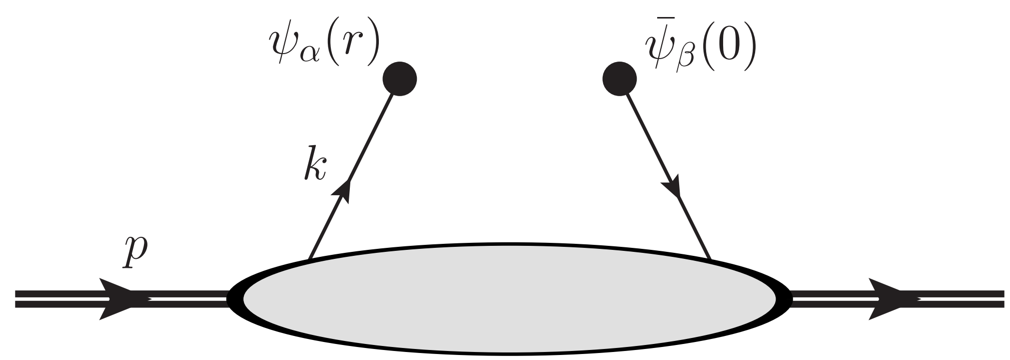


Counting the Number of Quarks

<u>The simplest thing:</u> Number operator in a hadronic state

$$E_k \frac{dN_\sigma}{d^3k} = \frac{1}{2(2\pi)^3} \frac{1}{2\Omega} \langle h(p)| \, b^\dagger_{k\sigma} b_{k\sigma} \, |h(p)\rangle$$

Volume factor normalizes plane-wave states



In terms of the quark fields:

$$\int d^3r \, e^{-i\vec{k}\cdot\vec{r}} \langle h(p)| \, \underline{\bar\psi_\beta(0)\, \psi_\alpha(r)} \, |h(p)\rangle \sim$$

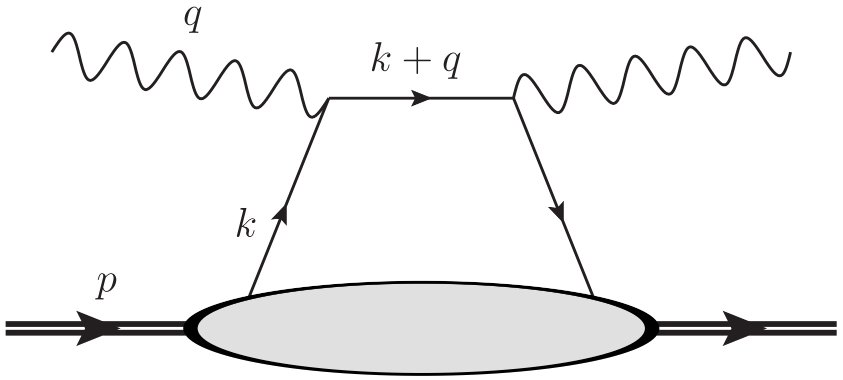
$$\sim \sum_{\sigma\lambda} \langle h(p)| \, b^\dagger_{k\sigma} b_{k\lambda} \, |h(p)\rangle \;\; \underline{\left[\bar U_\sigma(k)\right]_\beta \left[U_\lambda(k)\right]_\alpha}$$

Dirac indices in the fields
→ Spin of the quarks

The Parton Model of DIS

The quark distribution is always a part of a larger process, like Deep Inelastic Scattering.

$$e + h \to e + X$$

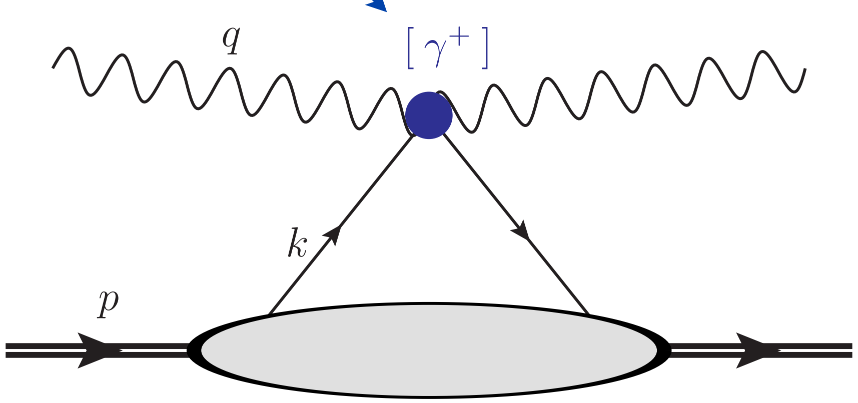


In DIS with Bjorken kinematics,

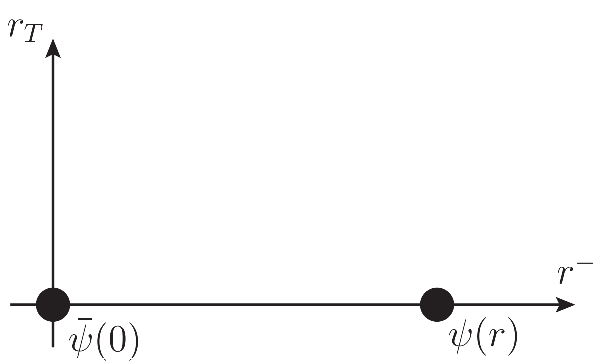
$$Q^2, s \to \infty \qquad\qquad x_B = \frac{Q^2}{s+Q^2} = const$$

the struck quark moves at the speed of light along the x^- axis.

vertex for unpolarized quarks



- ➡ DIS measures a one-dimensional distribution of quarks $\frac{dN}{dx}$
- ➡ Photon couples to the number of unpolarized quarks with an effective vertex γ^+
- ➡ The separation between the quark fields is lightlike along the x^- axis.



The Naive Quark Distribution

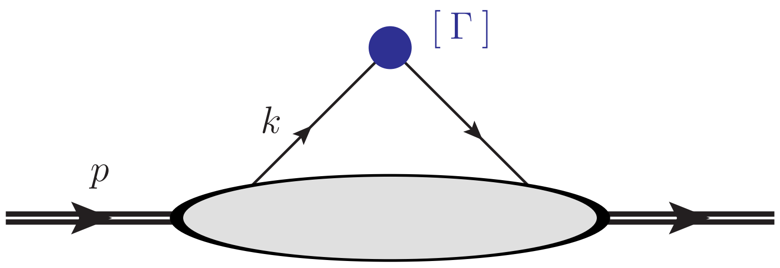
Collinear quark fields

$$\frac{dN}{dx} = \int \frac{dr^-}{2\pi} e^{ixp^+ r^-} \langle h(p) | \underline{\bar{\psi}(0) \frac{\gamma^+}{2} \psi(r)} | h(p) \rangle$$

$$= \frac{1}{2(2\pi)^3} \frac{1}{2\Omega} \frac{1}{x^2 p^+} \sum_{\sigma\lambda} \int d^2k \, \langle h(p) | b^\dagger_{k\sigma} b_{k\lambda} | h(p) \rangle \, \underline{\left[\bar{U}_\sigma(k) \frac{\gamma^+}{2} U_\lambda(k) \right]}$$

Vertex selects quark polarization

Other effective vertices Γ can couple to different quark spins: (e.g., νDIS)



γ^+ = Unpolarized: $\bar{U}_\sigma(k) \gamma^+ U_\lambda(k) = 2xp^+ [\mathbf{1}]_{\sigma\lambda}$

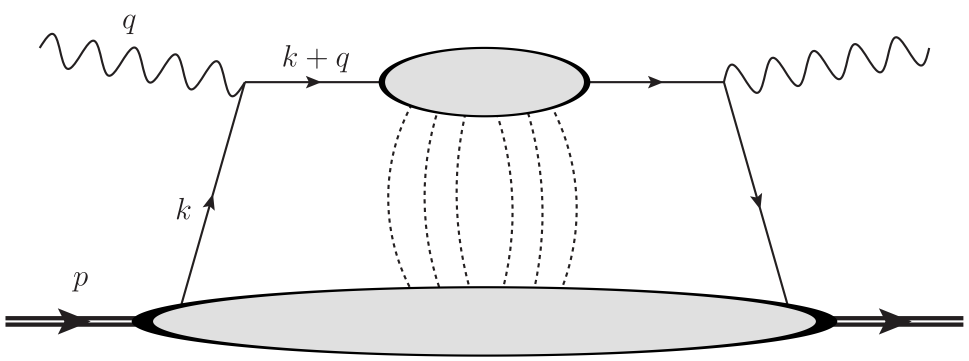
$\gamma^+ \gamma^5$ = Longitudinal: $\bar{U}_\sigma(k) \gamma^+ \gamma^5 U_\lambda(k) = 2xp^+ [\sigma^3]_{\sigma\lambda}$

$\gamma^+ \gamma^j_\perp \gamma^5$ = Transverse: $\bar{U}_\sigma(k) \gamma^+ \gamma^j_\perp \gamma^5 U_\lambda(k) = 2xp^+ [\sigma^j_\perp]_{\sigma\lambda}$

Gauge Invariance

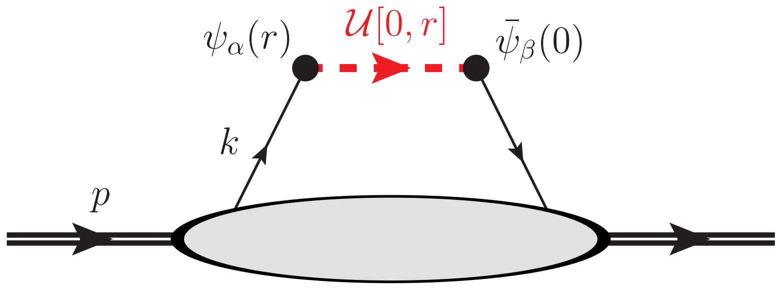
The naive quark number operator is not gauge invariant:

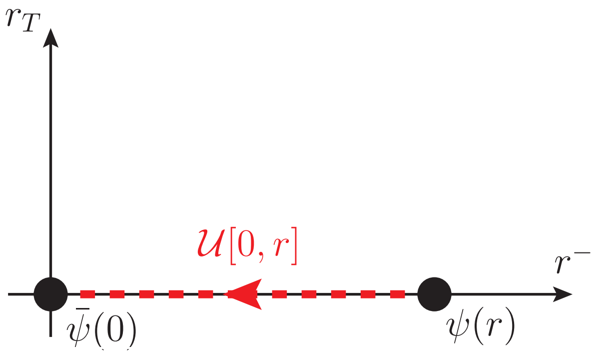
$$\bar{\psi}_\beta(0)\,\psi_\alpha(r) \to \bar{\psi}_\beta(0)\,S^{-1}(0)\,S(r)\,\psi_\alpha(r)$$



The struck quark is not free; it moves in the gauge field of the target:

$$\mathcal{U}[0,r] = \mathcal{P}\exp\left[i\int_{r^-}^{0^-} dz^- T^a A^{+a}(0^+, z^-, 0_\perp)\right]$$





The direction is fixed by factorization of the quark distribution from the physical process.

The dressed operator is gauge invariant: $\bar{\psi}_\beta(0)\,\mathcal{U}[0,r]\,\psi_\alpha(r)$

➡ ...but it is no longer purely a quark operator.

Collinear Quark Distribution Functions

The proper gauge-invariant quark correlator is

$$\phi_{\alpha\beta}(x) = \int \frac{dr^-}{2\pi} e^{ixp^+r^-} \langle h(p,S)| \, \bar{\psi}_\beta(0) \, \mathcal{U}[0,r] \, \psi_\alpha(r) \, |h(p,S)\rangle$$

from which we can project the distributions of polarized quarks:

$$\tfrac{1}{2}\mathrm{Tr}[\phi\gamma^+] = f_1(x)$$ Unpolarized Distribution

$$\tfrac{1}{2}\mathrm{Tr}[\phi\gamma^+\gamma^5] = S_L \, g_1(x)$$ Helicity Distribution

$$\tfrac{1}{2}\mathrm{Tr}[\phi\gamma^+\gamma_\perp^j\gamma^5] = S_\perp^j \, h_1(x)$$ Transversity Distribution

Is it still a distribution of quarks? Or is it contaminated by gluons?

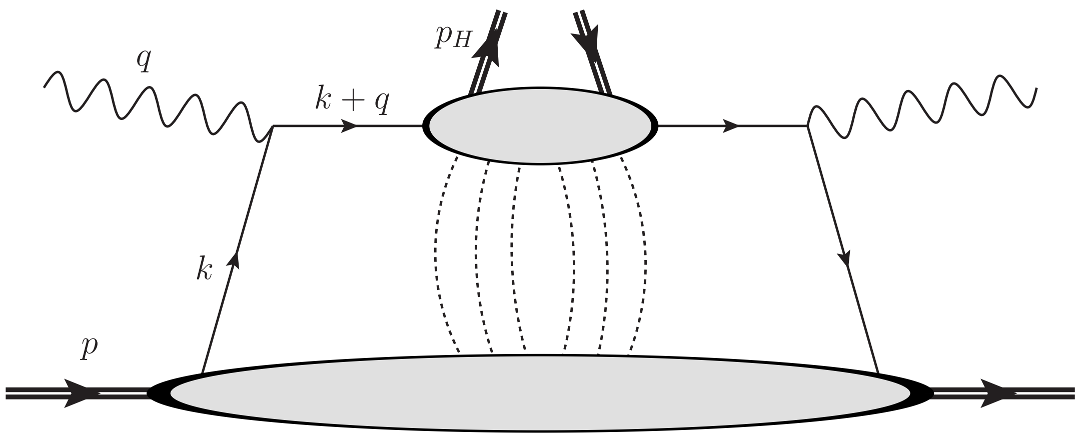
- The gauge link can be gauged away by choosing $A^+ = 0$

➡ Recover the naive quark number interpretation.

What About the Transverse Momentum?

For Semi-Inclusive Deep Inelastic Scattering (SIDIS), we can study the distribution as a function of transverse momentum.

$$e + h \to e + h' + X$$



- Sensitive to the transverse momentum dependence (TMD) in the quark distribution.
- Also sensitive to the TMD fragmentation process.

It's easy to define an "unintegrated quark distribution":

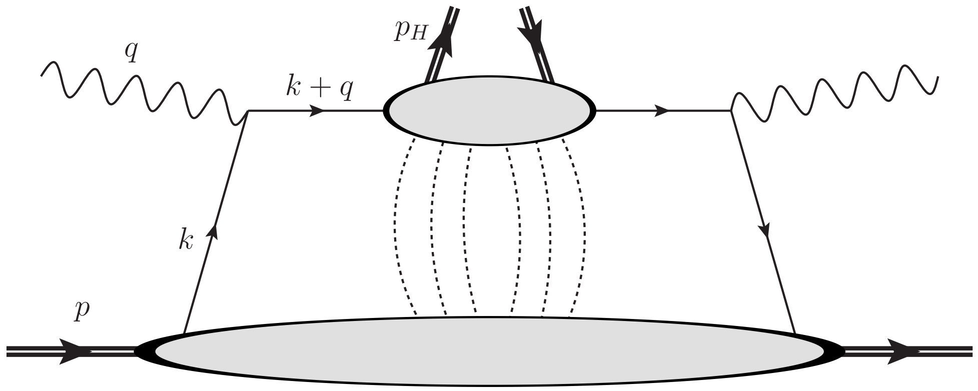
$$\phi_{\alpha\beta}(x) = \int d^2k \, \underline{\phi_{\alpha\beta}(x, \vec{k}_\perp)}$$

$$\phi_{\alpha\beta}(x, \vec{k}_\perp) = \int \frac{d^{2-}r}{(2\pi)^3} e^{ik\cdot r} \langle h(p)| \, \bar{\psi}_\beta(0) \, \mathcal{U}[0,r] \, \psi_\alpha(r) \, |h(p)\rangle$$

...but the machinery works very differently under the hood.

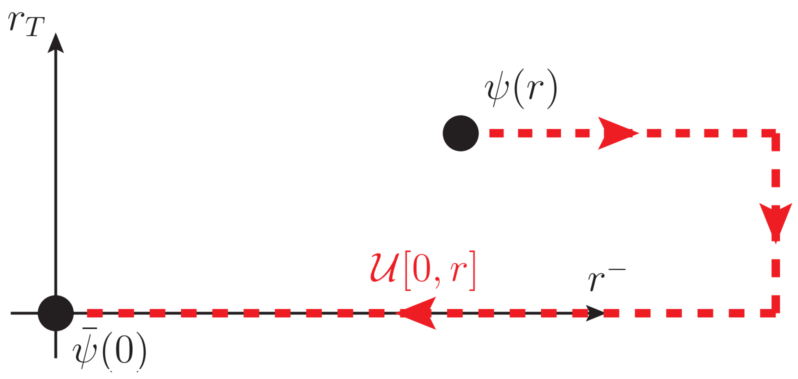
The Importance of the Glue

Fixing the transverse momentum separates the quark fields in the transverse plane.



The gauge link is now a highly nontrivial "staple":

- The "future-pointing" color flow is still fixed from the factorization of SIDIS.
- Because of the transverse separation, the gauge link is free to flow all the way to "light-cone infinity."
- The two light-like legs are connected by a transverse gauge link at infinity.



$$\mathcal{U}[0,r] = \mathcal{U}_{0_\perp}[0^-,\infty^-]\,\mathcal{U}_\perp[\vec{0}_\perp,\vec{r}_\perp]\,\mathcal{U}_{r_\perp}[\infty^-,r^-]$$

- The gauge link cannot be fully gauged away, even with $A^+ = 0$
- It carries physical information about the extra transverse momentum acquired from the color Lorentz force.

Burkardt, Phys. Rev. D88 (2013)

Non-Universality

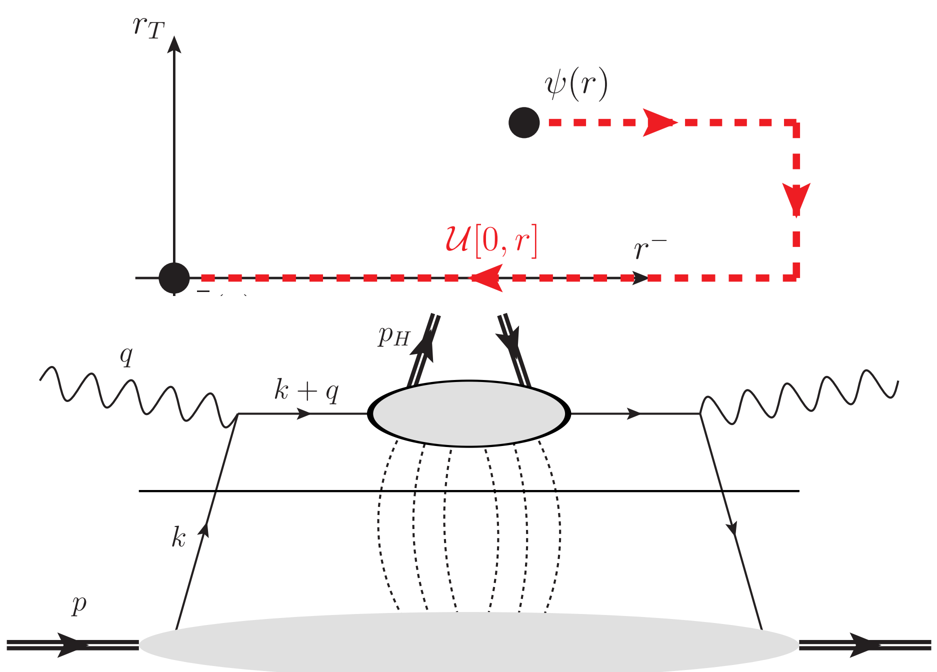
The dependence on the direction of the gauge link violates universality.

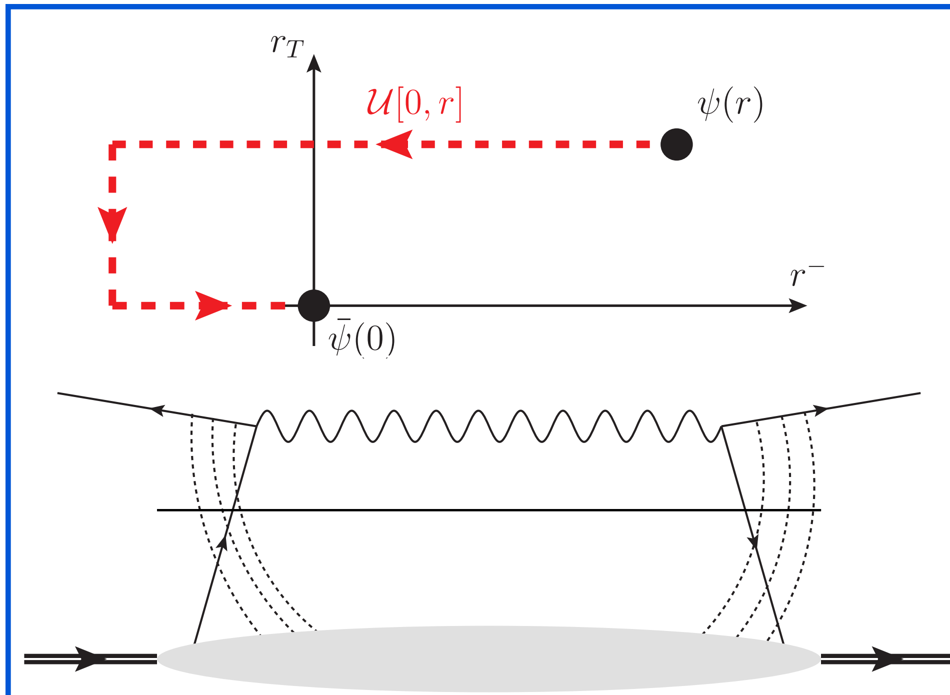
- PT symmetry, for example, is an exact symmetry of the collinear PDF's.
- But for the TMD distributions, PT symmetry alters the trajectory of the gauge link from future-pointing to past-pointing.
- The TMD distributions measured in Semi-Inclusive Deep Inelastic Scattering can differ by a sign from the ones measured in the Drell-Yan process.

SIDIS

Future-Pointing

Final-State Interactions





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Past-Pointing

Initial-State Interactions

The Quark TMD's

At leading order, there are 8 independent quark TMD parton distributions:

Unpolarized

Sivers function (PT-odd)

2 Unpolarized:
$$\frac{1}{2}\mathrm{Tr}[\phi\gamma^+] = f_1 - \frac{\vec{k}_\perp \times \vec{S}_\perp}{m} f_{1T}^\perp$$

Helicity

Worm-gear

2 Longitudinal:
$$\frac{1}{2}\mathrm{Tr}[\phi\gamma^+\gamma^5] = S_L g_1 + \frac{\vec{k}_\perp \cdot \vec{S}_\perp}{m} g_{1T}$$

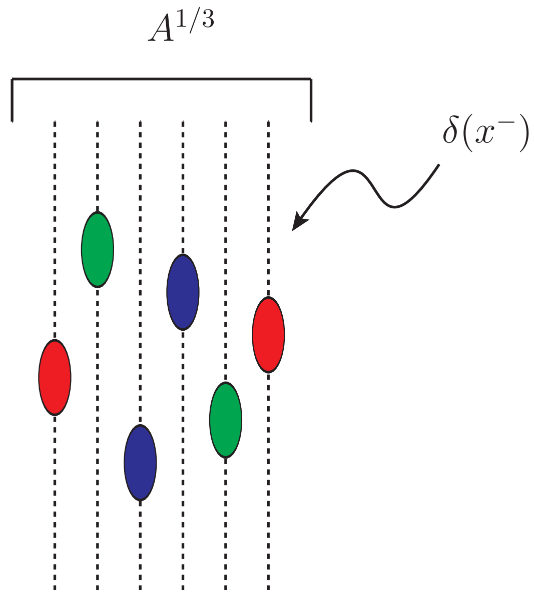
Worm-gear

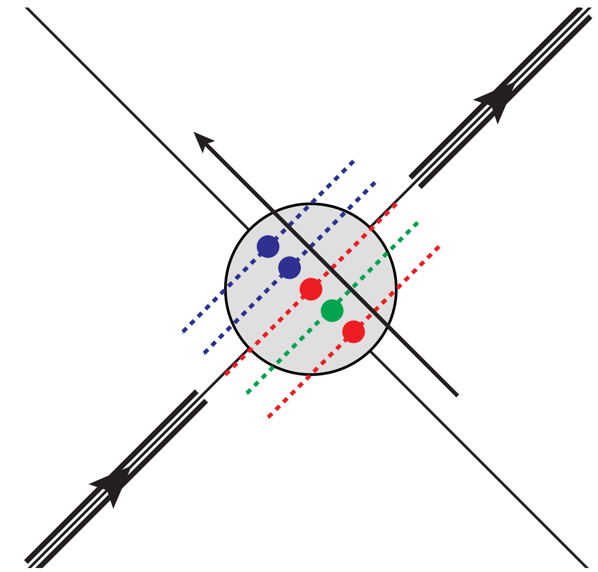
Transversity

Boer-Mulders function (PT-odd)

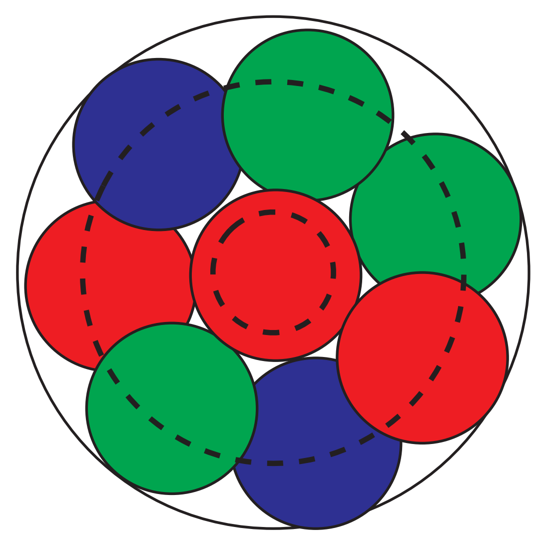
4 Transverse:
$$\frac{1}{2}\mathrm{Tr}[\phi\gamma^+\gamma_\perp^j\gamma^5] = S_\perp^j h_1 + \frac{k_\perp^j}{m} S_L h_{1L}^\perp + \epsilon_T^{ji} \frac{k_\perp^i}{m} h_1^\perp + \left(\frac{k_\perp^i k_\perp^j}{m^2} - \delta^{ij} \frac{k_T^2}{2m^2} \right) S_\perp^i h_{1T}^\perp$$

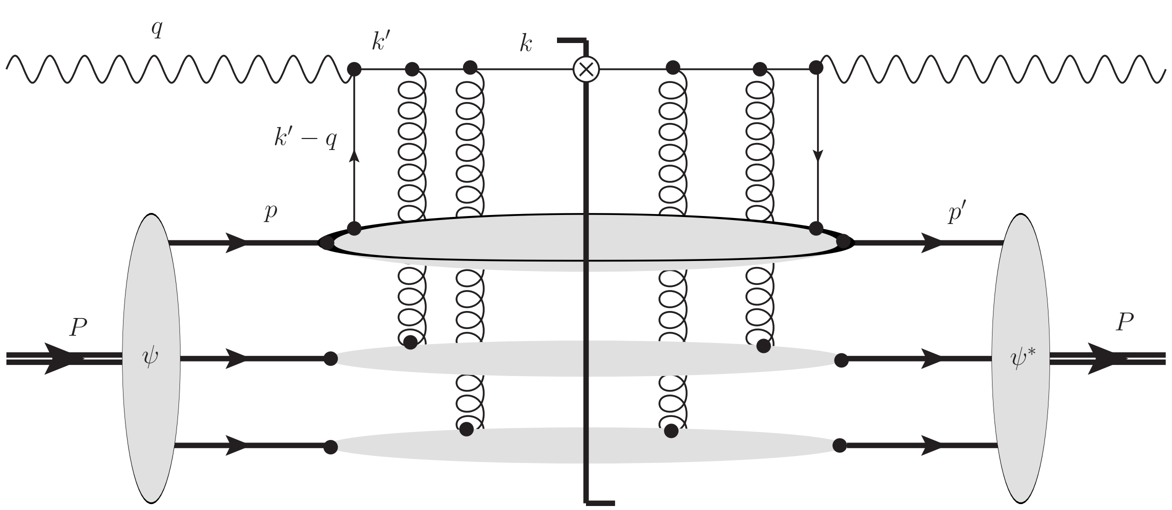
Pretzelosity

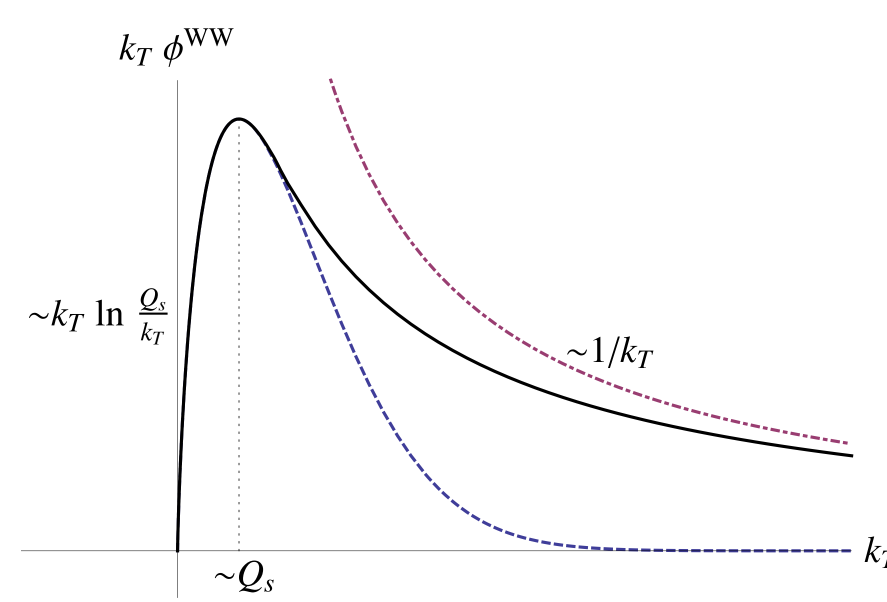




The Power of the High-Density Limit





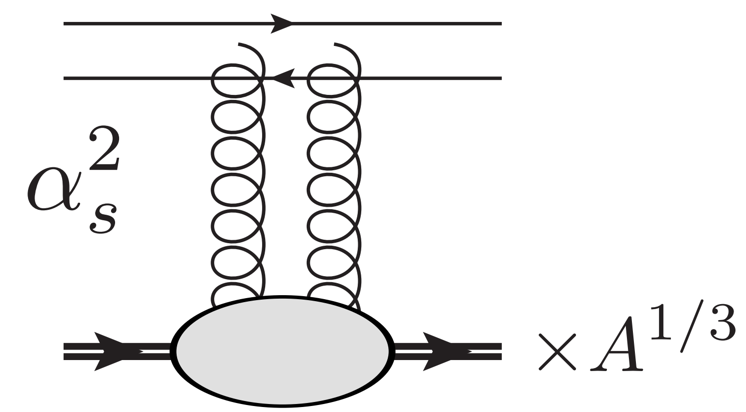


High Density, Classical Fields

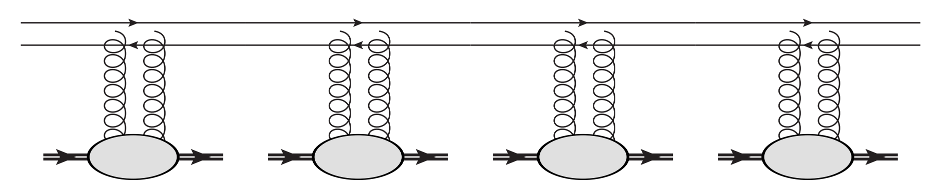
Consider a heavy nucleus with a large number of nucleons which moves high energy along the x^+ axis.

$$A \gg 1$$

The nucleus may have a low 3-dimensional density, but when the many nucleons are Lorentz-contracted, they generate a large 2-dimensional density.



A projectile has a low probability $\sim \alpha_s^2$ to interact with any nucleon, but this is enhanced by the large number $A^{1/3}$ of nucleons at a given impact parameter.

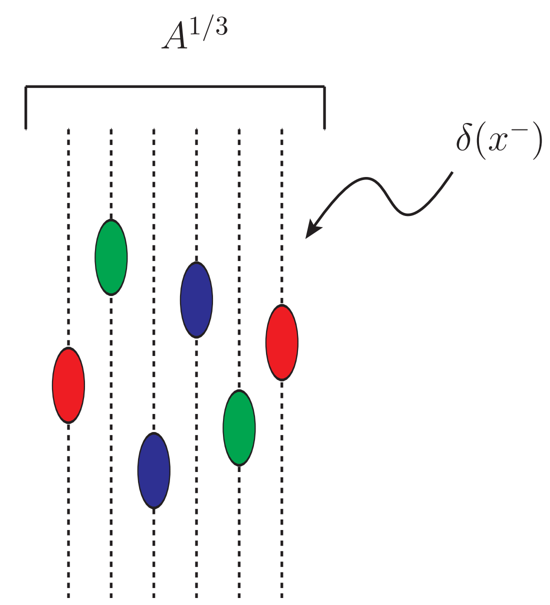


When $\alpha_s^2 A^{1/3} \sim \mathcal{O}(1)$, the interaction strength becomes $\mathcal{O}(1)$ and the projectile effectively propagates through the classical gluon field of the nucleus.

The McLerran-Venugopalan Model

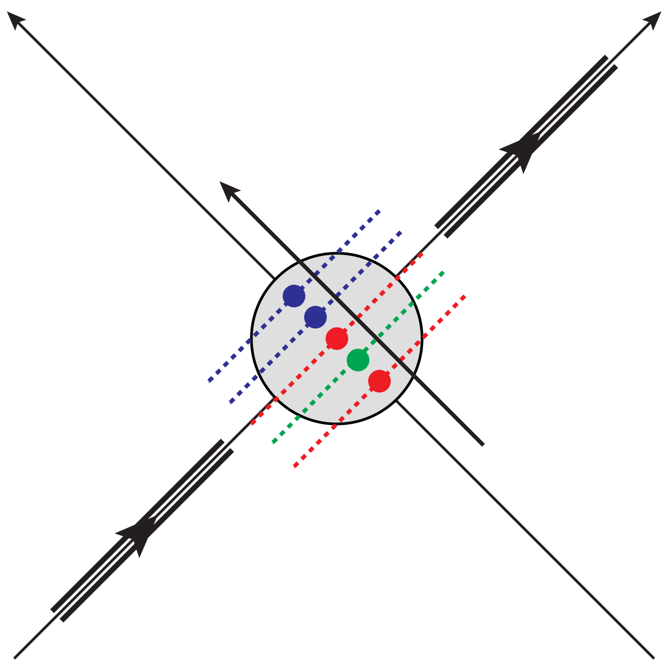
- In Feynman gauge, the classical field of each nucleon is localized along the x^- axis:

$$A^{+a}(x^+, x^-, \vec{x}_\perp) = \frac{g}{2\pi} T^a \delta(x^-) \ln(x_T \Lambda)$$



- The nucleons have a small but finite separation in x^-, so each nucleon's color field is generated independently.

- When the projectile crosses the nucleus, it undergoes a random walk in the color space of the nucleons and in the transverse momentum the nucleon field delivers.



- The typical transverse momentum a projectile acquires from crossing the nucleus defines the saturation scale Q_s

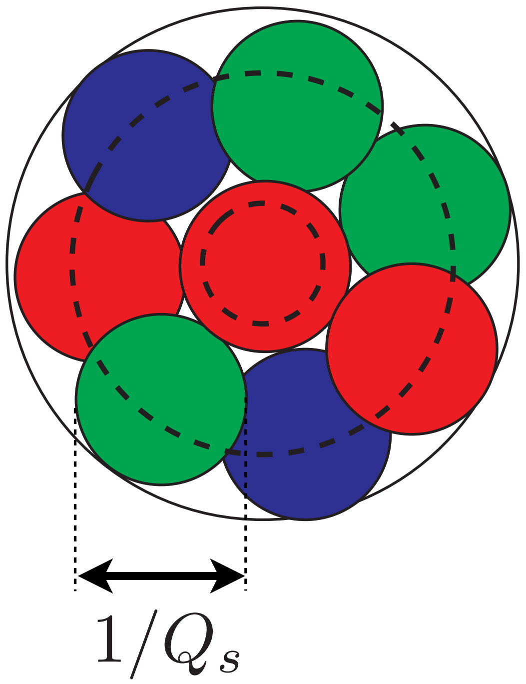
$$Q_s^2(\vec{b}_\perp) \propto \alpha_s^2 T(\vec{b}_\perp) \sim \alpha_s^2 A^{1/3} \Lambda^2$$

Gluon Saturation

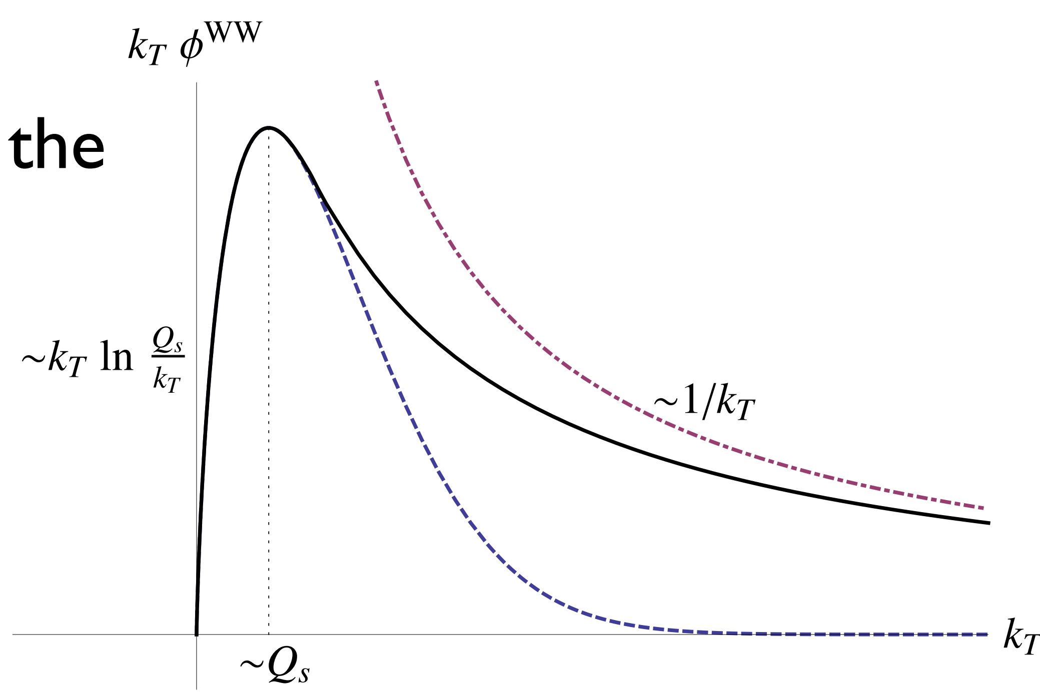
- The inverse saturation scale defines a correlation length in the transverse plane over which the color fields are correlated.

- The color fields over short distances is qualitatively different from the fields over longer distances:

➡ At short distances (large transverse momentum), the gluon field is correlated and matches the field of a single color source.



➡ Over long distances (low transverse momentum), the gluon field is uncorrelated and screened.



- The saturation scale dynamically cuts off the gluon distribution in the IR.

- If the charge density is high enough that $Q_s^2 \gg \Lambda^2$ then the process can be calculated perturbatively.

The Power of the High-Density Limit

Can we use the high-density quasi-classical limit to simplify the TMD quark correlator?

- Quark correlator of a heavy nucleus in the MV model:

$$\Phi_{\alpha\beta}(x, \vec{k}_\perp) = \int \frac{d^{2-}r}{(2\pi)^3} e^{ik\cdot r} \langle A(P)| \, \bar{\psi}_\beta(0) \, \mathcal{U}[0,r] \, \psi_\alpha(r) \, |A(P)\rangle$$

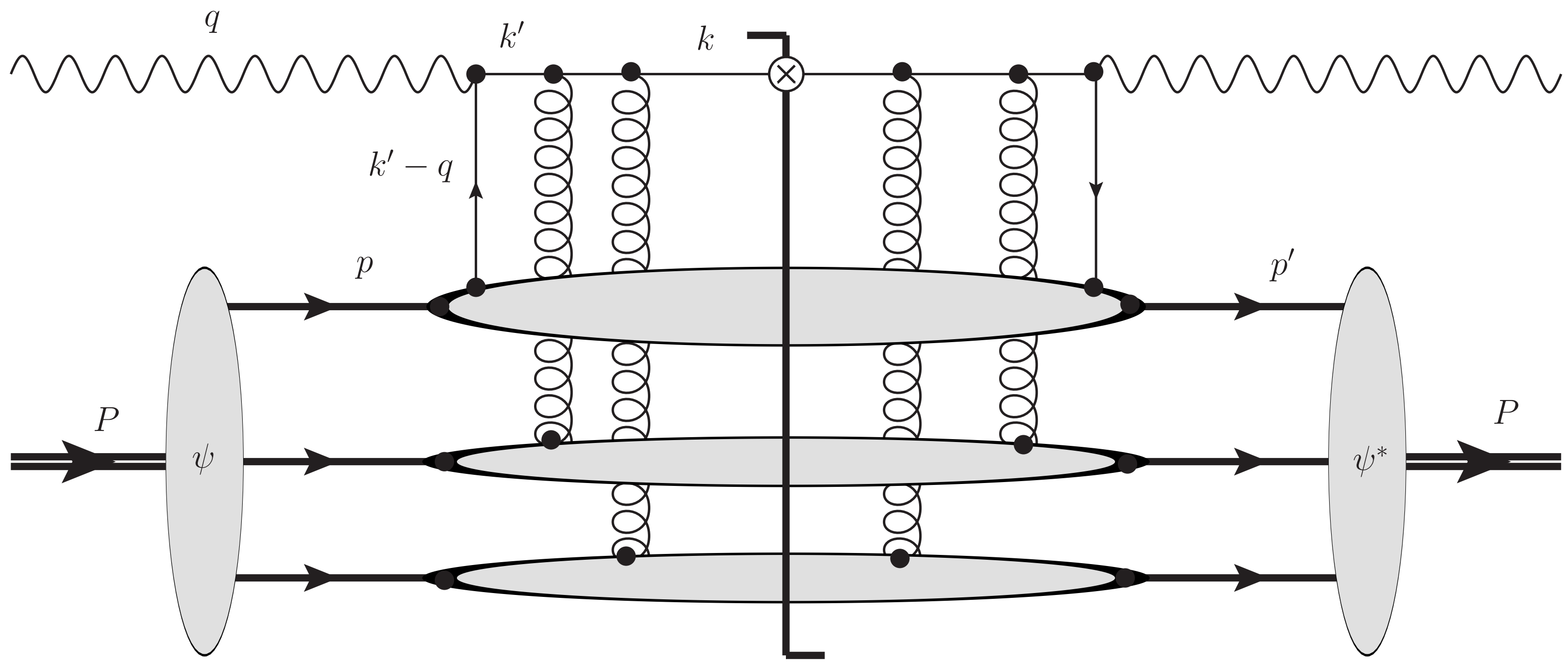
Regard the nucleus as a distribution of nucleons with some light-front wave function:

$$|A(P)\rangle = \int d\Omega \, \underline{\Psi_N(p_1, \cdots, p_n)} \, |N_1(p_1) \cdots N_A(p_A)\rangle$$

If the quark field acts on one nucleon $|N(p)\rangle$, the rescattering takes place predominantly on the other $(A-1)$ spectator nucleons.

- Up to corrections of $\mathcal{O}(\alpha_s^2) \sim \mathcal{O}(A^{-1/3})$ it is possible to separate the wave function of the nucleons, the quark distribution of a nucleon, and the perturbatively calculable gauge link!

The Pieces of the Puzzle



$$\langle A | \, \bar{\psi}_\beta(0) \, \mathcal{U}[0,r] \, \psi_\alpha(r) \, | A \rangle \approx$$

$$\approx \int d\Omega d\Omega' \, \underline{\Psi_N(\Omega) \Psi_N^*(\Omega')}$$

Light-front wave functions of the nucleons

$$\times \, \underline{\langle N(p') | \, \bar{\psi}_\beta(0) \, u[0,r] \, \psi_\alpha(r) \, | N(p) \rangle}$$

Quark correlator of a nucleon up to $\mathcal{O}(\alpha_s)$

$$\times \, \underline{\langle A - 1 | \, \mathcal{U}[0,r] \, | A - 1 \rangle}$$

Gauge link calculated in the MV model

Quasi-Classical Factorization

$$\Phi_{\alpha\beta}(x, \vec{k}_\perp) = \frac{A}{(2\pi)^5} \sum_{\sigma\sigma'} \int d^{2+}p \, d^{2-}b \, d^2 r \, d^2 k' \, e^{-i(\vec{k}_\perp - \vec{k}'_\perp - \hat{x}\vec{p}_\perp)\cdot \vec{r}_\perp}$$

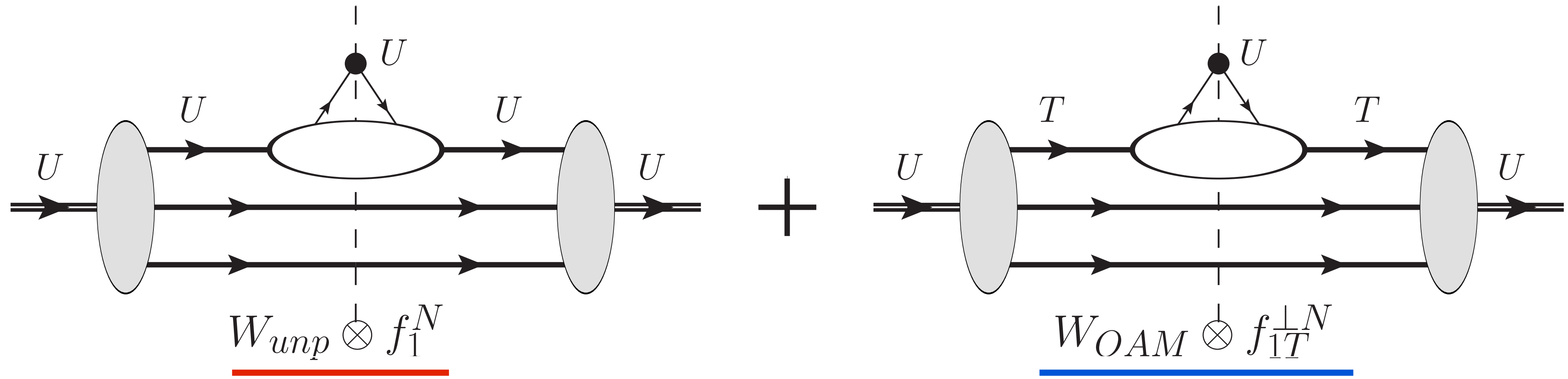
$$\times \underbrace{W_{\sigma'\sigma}(p, b)}_{\text{Wigner distribution of nucleons}} \, \underbrace{[\phi^N_{\alpha\beta}(\hat{x}, \vec{k}'_\perp)]_{\sigma,\sigma'}}_{\text{Nucleonic TMD}} \, \underbrace{S^{[\infty^-, b^-]}_{(r_T, b_T)}}_{\text{Gauge Link}}$$

$$\hat{x} = \frac{P^+}{p^+} x$$

$$\boxed{W_{\sigma'\sigma}(p, b) = \frac{1}{2(2\pi)^3} \int \frac{d^{2+}(p-p')}{\sqrt{p^+ p'^+}} \, e^{-i(p-p')\cdot b} \, \Psi^N_\sigma(p) \, \Psi^{N*}_{\sigma'}(p')}$$

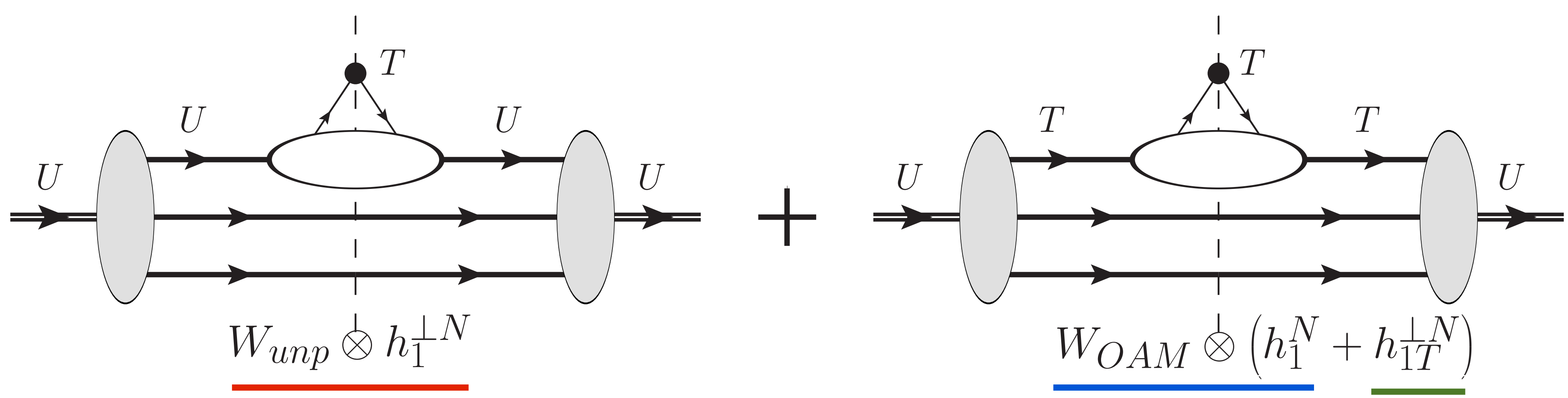
$$\boxed{S^{[\infty^-, b^-]}_{(r_T, b_T)} = \exp\left[-\frac{1}{4} r_T^2 Q_s^2(b_T) \left(\frac{R^-(b_T) - b^-}{2R^-(b_T)} \right) \right]}$$

$f_1^A =$

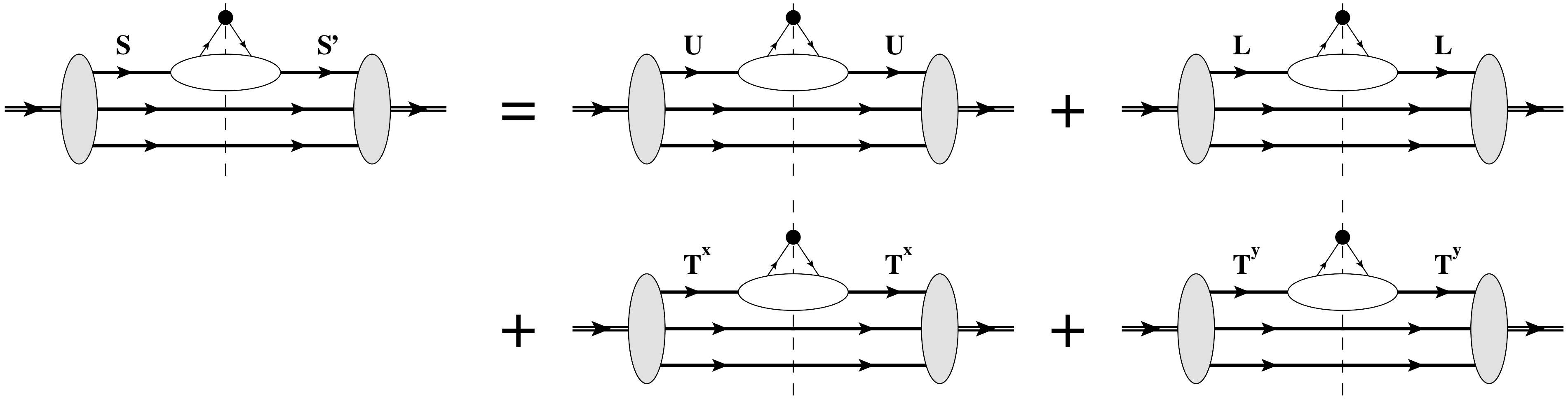


Spin-Orbit Coupling in an Unpolarized Nucleus

$h_1^{\perp A} =$



Polarized Nucleons in an Unpolarized Nucleus



The Wigner distribution $W_{\sigma'\sigma}$ and nucleonic quark correlator $\phi_{\sigma\sigma'}$ are (2×2) spin density matrices.

- In the nucleon rest frame, they can be expanded in a basis of Pauli matrices and the unit matrix:

$$W_{\sigma'\sigma} = W_{unp} \, \underline{[\mathbf{1}]_{\sigma'\sigma}} + \vec{W}_{pol} \cdot \underline{[\vec{\sigma}]_{\sigma'\sigma}}$$

$$W_{unp} = \frac{1}{2} \mathrm{Tr}[W]$$

$$\vec{W}_{pol} = \frac{1}{2} \mathrm{Tr}[W \, \vec{\sigma}]$$

With these components, you can construct a nucleon in any spin state:

$$W(p, b, S) = W_{unp}(p, b) + \vec{S} \cdot \vec{W}_{pol}(p, b)$$

This expansion makes the nucleon spin state transparent: it can either be unpolarized (U), longitudinally-polarized (L), or transversely-polarized (T).

Lorentz-Covariant Spin Structure

In the nucleon rest frame, the trace over spin indices becomes a sum over the 4 independent spin configurations (U, L, T^x, T^y):

$$\frac{1}{2} W_{\sigma' \sigma} \phi_{\sigma \sigma'} = W_{unp} \phi_{unp} + \vec{W}_{pol} \cdot \vec{\phi}_{pol}$$

You can generalize S to a four-vector and use it to boost these expressions out of the rest frame:

$$S^\mu = (0, \vec{S}) \to \left(S_L \frac{p^+}{m}, -S_L \frac{p^-}{m}, \vec{S}_\perp \right)$$

$$W(p, b, S) = W_{unp}(p, b) - S_\mu W^\mu_{pol}(p, b)$$

$$\frac{1}{2} W_{\sigma' \sigma} \phi_{\sigma \sigma'} = W_{unp} \phi_{unp} - W_{pol\,\mu} \phi^\mu_{pol}$$

Then the quasi-classical factorization formula becomes:

$$\Phi_{\alpha\beta}(x, \vec{k}_\perp) = \frac{2A}{(2\pi)^5} \int d^{2+}p \, d^{2-}b \, d^2 r \, d^2 k' \, e^{-i(\vec{k}_\perp - \vec{k}'_\perp - \hat{x}\vec{p}_\perp) \cdot \vec{r}_\perp}$$
$$\times \left(W_{unp}(p, b) \phi_{unp}(\hat{x}, \vec{k}'_\perp) - W_{pol\,\mu}(p, b) \phi^\mu_{pol}(\hat{x}, \vec{k}'_\perp) \right) S^{[\infty^-, b^-]}_{(r_T, b_T)}$$

Symmetries of the Nucleus

$$W_{\sigma'\sigma}(p,b) = \frac{1}{2(2\pi)^3} \int \frac{d^{2+}(p-p')}{\sqrt{p^+ p'^+}} \, e^{-i(p-p')\cdot b} \, \Psi_\sigma^N(p) \, \Psi_{\sigma'}^{N*}(p')$$

Since the Wigner distribution is built from only light-front wave functions, it has a high degree of symmetry:

- Discrete symmetries like PT
- No dependence on the collision axis (virtual photon)
- Should possess full 3D rotational symmetry in the rest frame
- Gets integrated with other factors possessing 2D rotational symmetry about the beam axis (virtual photon)

Using all these symmetries, we should be able to strongly constrain the functional form of the Wigner distribution.

- What kind of spin-orbit coupling is permitted by these symmetries?

... but there's a catch.

Covariant Light-Front Perturbation Theory

Light-front wave functions are quantized at fixed "light-front time" $x^+ = ct + z$

- Even though they don't depend on the collision axis, they do have a built in preferred axis of their own (z)
- These wave functions are optimized for describing high-energy states with a preferred collision axis: boost-invariant, 2D rotationally invariant, etc.
- 3D rotations are "dynamical": they couple to the interaction Hamiltonian, changing the particle content of the state and requiring an exact solution.

A proper description of rotations in the light-front formalism requires "covariant light-front perturbation theory"

Carbonell, et. al, Phys. Rept. 300 (1998)

- Keeps the quantization axis arbitrary instead of using the z axis .
- To preserve Lorentz covariance, you must rotate the quantization axis as well!
- In general, relativistic LFWF depend on the direction of the quantization axis.

➡ They do not possess 3D rotational invariance in the kinematic variables....

Nucleons with Non-Relativistic Motion

But in the non-relativistic limit $c \to \infty$, the light-front quantization condition reduces down to the equal-time quantization condition:

Carbonell, et. al, Phys. Rept. 300 (1998)

$$(ct + \vec{x} \cdot \hat{n} = const) \;\rightarrow\; (ct = const)$$

➡ Nonrelativistic LFWF are equivalent to equal-time WF, which have no dependence on the special direction $\hat{n}$.

If the nucleons move non-relativistically in the nucleus, then their WF do possess 3D rotational invariance in the nuclear rest frame!

In the non-relativistic limit:

$$W_{\sigma' \sigma}(\vec{p}, \vec{b}) = \frac{1}{2(2\pi)^3 m} \int d^3(p - p')\, e^{+i(\vec{p}-\vec{p}')\cdot\vec{b}}\, \Psi_\sigma^N(\vec{p}^{\,2})\, \Psi_{\sigma'}^{N*}(\vec{p}^{\,\prime 2})$$

where the vector quantities are

$$\underline{\vec{p} = \left(\vec{p}_\perp \, , \, (Am)(\tfrac{p^+}{P^+} - \tfrac{1}{A})\right)} \qquad \underline{\vec{b} = \left(\vec{b}_\perp \, , \, -\frac{P^+ b^-}{Am}\right)}$$

Parameterizing the Wigner Distribution

From 3D rotational invariance, parity, and time-reversal invariance:

$$W(\vec{p}, \vec{b}, \vec{S}) = W_{unp}[\vec{p}^{\,2}, \vec{b}^{\,2}, (\vec{p}\cdot\vec{b})^2] + \underline{\vec{S}\cdot(\vec{b}\times\vec{p})}\, W_{OAM}[\vec{p}^{\,2}, \vec{b}^{\,2}, (\vec{p}\cdot\vec{b})^2]$$

$(\vec{L}\cdot\vec{S})$ spin-orbit coupling!

The Wigner distribution is integrated over impact parameters with the gauge factor, which possesses 2D rotational invariance:

$$\int d^2b\, W(\vec{p}, \vec{b}, \vec{S})\, S(b_T)$$

- Without loss of generality, we can replace $b_\perp^i b_\perp^j \rightarrow \frac{1}{2} b_T^2 \delta^{ij}$

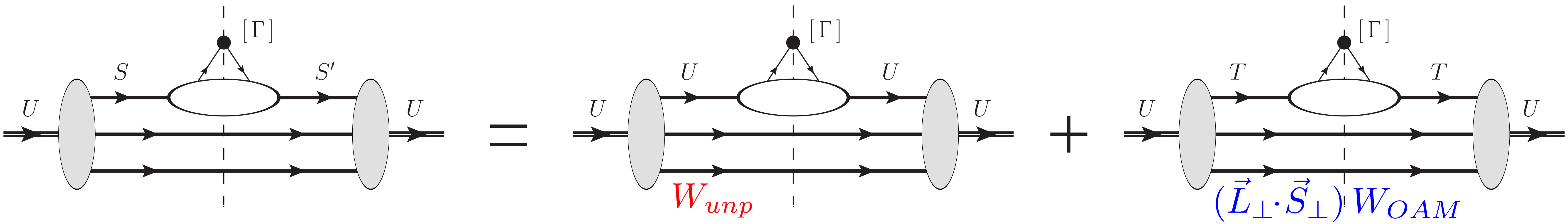
The maximum spin-orbit structure of an unpolarized nucleus is then:

$$\boxed{W(\vec{p}, \vec{b}, \vec{S}) \Rightarrow W_{unp}[p_T^2, b_T^2; p_z^2, b_z^2] - b_z\, (\vec{p}_\perp \times \vec{S}_\perp)\, W_{OAM}[p_T^2, b_T^2; p_z^2, b_z^2]}$$

and we have the dictionary

$$\boxed{p_z = (Am)\left(\frac{p^+}{P^+} - \frac{1}{A}\right) \qquad\qquad b_z = -\frac{P^+ b^-}{Am}}$$

Spin-Orbit Structure in the Quark Distribution



$$W(\vec{p}, \vec{b}, \vec{S}) \Rightarrow W_{unp}[p_T^2, b_T^2; p_z^2, b_z^2] - b_z\, (\vec{p}_\perp \times \vec{S}_\perp)\, W_{OAM}[p_T^2, b_T^2; p_z^2, b_z^2]$$

In an unpolarized nucleus, the intermediate nucleons can only be unpolarized or transversely polarized

- Longitudinal polarizations do not survive the impact parameter integral

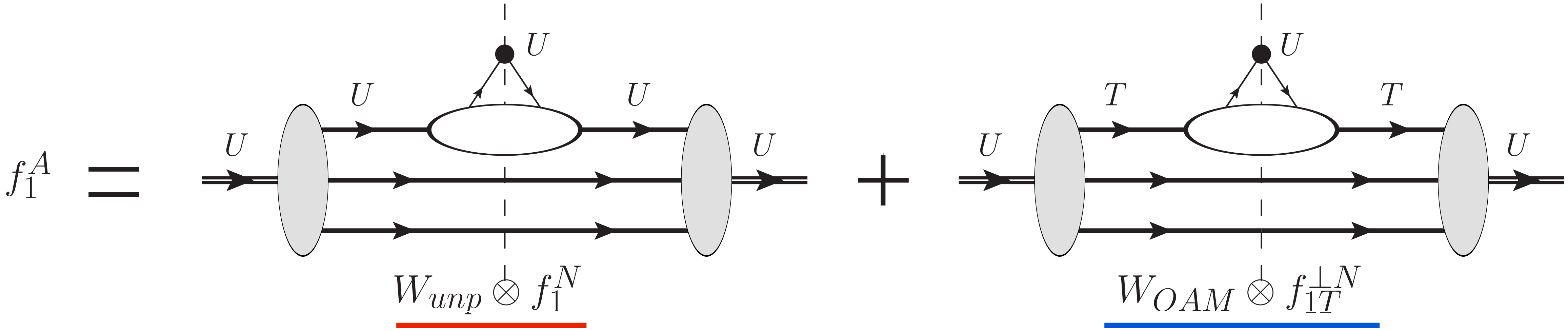
Only two leading-twist TMD quark distributions exist for an unpolarized nucleus:

$$\tfrac{1}{2}\mathrm{Tr}[\Phi\, \gamma^+] = \underline{f_1^A}$$ ⟵ Unpolarized quark distribution

$$\tfrac{1}{2}\mathrm{Tr}[\Phi\, \gamma^+ \gamma^5] = 0$$ ⟵ No longitudinally-polarized quarks

$$\tfrac{1}{2}\mathrm{Tr}[\Phi\, \gamma^+ \gamma_\perp^j \gamma^5] = \epsilon_T^{ji} \frac{k_\perp^i}{Am} \underline{h_1^{\perp A}}$$ ⟵ Boer-Mulders function: (PT)-odd quark spin-orbit coupling

Unpolarized Quark Distribution



$$f_1^A(x, k_T) = \frac{2A}{(2\pi)^5} \int d^{2+}p\, d^{2-}b\, d^2r\, d^2k'\, e^{-i(\vec{k}_\perp - \vec{k}'_\perp - \hat{x}\vec{p}_\perp)\cdot\vec{r}_\perp}\, S_{(r_T, b_T)}^{[\infty^-, b^-]}$$

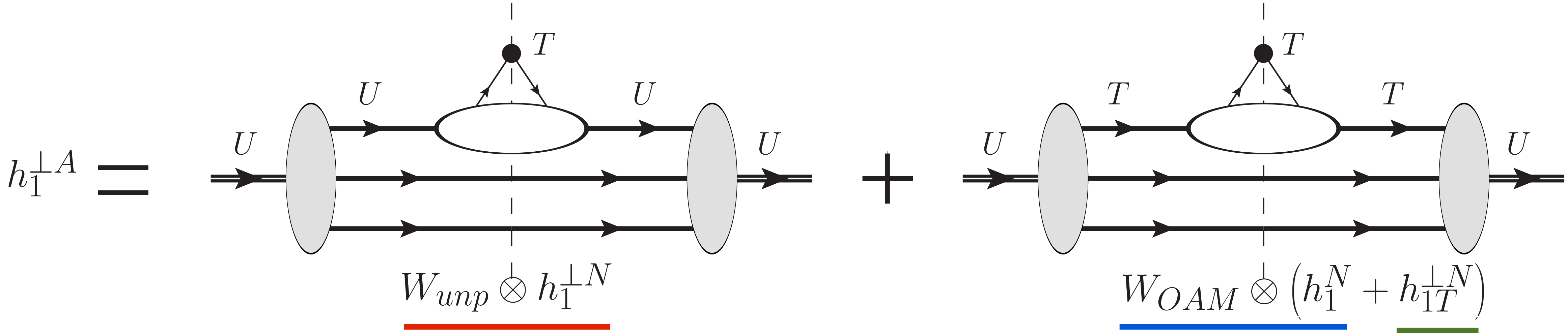
$$\times \left(\underline{W_{unp}(p,b) f_1^N(\hat{x}, k'_T)} - \underline{\frac{P^+ b^-}{A m^2} (\vec{p}_\perp \cdot \vec{k}'_\perp) W_{OAM}(p,b) f_{1T}^{\perp N}(\hat{x}, k'_T)} \right)$$

One channel builds up the unpolarized quark distribution of the nucleons:

$$f_1^N \to f_1^A \qquad (W_{unp})$$

A second channel generates transversely polarized nucleons with OAM, and their Sivers function builds up the unpolarized quark distribution:

$$f_{1T}^{\perp N} \to f_1^A \qquad (W_{OAM})$$



$$
h_1^{\perp A}(x, k_T) = \frac{2A}{(2\pi)^5} \frac{Am}{k_T^2} \int d^{2+}p \, d^{2-}b \, d^2 r \, d^2 k' \, e^{-i(\vec{k}_\perp - \vec{k}'_\perp - \hat{x}\vec{p}_\perp)\cdot \vec{r}_\perp} \, S^{[\infty^-, b^-]}_{(r_T, b_T)}
$$

$$
\times \Bigg(\frac{\vec{k}_\perp \cdot \vec{k}'_\perp}{m} W_{unp}(p,b) h_1^{\perp N}(\hat{x}, k'_T) - \frac{P^+ b^-}{Am} (\vec{p}_\perp \cdot \vec{k}_\perp) W_{OAM}(p,b) h_1^N(\hat{x}, k'_T)
$$

$$
- \frac{P^+ b^-}{Am} \left(\frac{(\vec{p}_\perp \times \vec{k}'_\perp)(\vec{k}_\perp \times \vec{k}'_\perp)}{m^2} - \frac{k_T'^2 \, (\vec{p}_\perp \cdot \vec{k}_\perp)}{2m^2} \right) W_{OAM}(p,b) h_{1T}^{\perp N}(\hat{x}, k'_T) \Bigg)
$$

One channel builds up the Boer-Mulders function of the nucleons:

$$
h_1^{\perp N} \to h_1^{\perp A} \quad (W_{unp})
$$

Another channel generates transversely polarized nucleons with OAM, and their transversity or pretzelosity build up the Boer-Mulders function:

$$
\left.\begin{array}{l} h_1^N \to h_1^{\perp A} \\ h_{1T}^{\perp N} \to h_1^{\perp A} \end{array}\right. \quad (W_{OAM})
$$

OAM and TMD Mixing

The presence of $(\vec{L} \cdot \vec{S})$ spin-orbit coupling induces nontrivial mixing between the nuclear and nucleonic TMD's.

- Fundamentally different from other authors, who only have p_T broadening effects. *Liang, et. al, Phys. Rev. D77 (2008)*
- The mixing occurs between the PT - even and PT - odd sectors:

PT - even

$$\boxed{f_1^A} \sim (P^+ b^-) W_{OAM} \otimes S_{(r_T, b_T)}^{[\infty^-, b^-]} \otimes \boxed{f_{1T}^{\perp N}}$$

PT - odd

PT - odd

$$h_1^{\perp A} \sim (P^+ b^-) W_{OAM} \otimes S_{(r_T, b_T)}^{[\infty^-, b^-]} \otimes h_1^N$$

$$h_1^{\perp A} \sim (P^+ b^-) W_{OAM} \otimes S_{(r_T, b_T)}^{[\infty^-, b^-]} \otimes h_{1T}^{\perp N}$$

PT - even

PT - reversing gauge factor

The mixing depends directly on the multiple rescattering on the spectator nucleons.

- If the gauge factor is replaced by unity, the mixing vanishes:

$$\int_{-\infty^-}^{\infty^-} db^- \, b^- \, W_{OAM}\left((b^-)^2\right) = 0$$

- OAM provides the spin-orbit coupling; the gauge link provides the PT breaking.

Implications for an EIC

A measurement of the p_T dependence of the nuclear TMD's which deviates from simple broadening of the corresponding nucleonic TMD is an indication of OAM.

- If f_1^N and $f_{1T}^{\perp N}$ are known, and f_1^A is measured, then the deviation of the nuclear distribution from the nucleonic one is directly proportional to W_{OAM}.
- It would require extensive p_T coverage, but in principle such measurements are possible at a future Electron-Ion Collider (EIC).

The same spin-orbit coupling $(\vec{L}_\perp \cdot \vec{S}_\perp) W_{OAM}$ is also responsible for the admixture of the transversity and pretzelosity into the nuclear Boer-Mulders function.

- Once W_{OAM} is measured from the admixture of the Sivers function into the unpolarized quark distribution, this provides a prediction for the amount of admixture present in the nuclear Boer-Mulders function.
- In this way, measuring the mixing of TMD's provides direct access to the orbital angular momentum present in the nucleus.

Assumptions and Context

Any kind of spin-orbit coupling, together with a dense medium, generically leads to TMD mixing of this kind.

The assumptions that lead to the possibility of TMD mixing required only the high-density limit and non-relativistic nucleon motion.

- The high-density limit is a genuine resummation of QCD. It should be valid not only for a heavy nucleus, but for any hadronic system at high energies.
- The mixing present in a dense, non-relativistic system should also be present in a dense relativistic system such as a high-energy proton. There may also be additional mixing which is not present in the non-relativistic case.
- All of the real model dependence resides in the structure of the Wigner distribution, which is highly constrained by symmetry.

In a similar manner, one can imagine constructing the TMD's of a dense proton from the calculated TMD's of its valence quarks. The proton should be highly relativistic and contain more structures than appeared here.

Outlook and Ongoing Analysis

- Use simple models for the Wigner distribution (e.g. static MV model, Gaussian distribution, etc.) to generate analytic curves for the form of the TMD's with or without the presence of OAM.

- Add to this the explicit TMD's of a quark target to build up a fully analytic form for the TMD's of the nucleus, using only ingredients obtained from QCD.

  ➡ By varying the few parameters of the model (effective masses, charges, Q_s) this functional form may be useful for fitting the TMD's of the dense proton.

- Apply this methodology to all the leading-twist TMD's of the heavy nucleus.

  ➡ A small number of spin-spin and spin-orbit coupling terms in the Wigner distribution will be responsible for a large number of mixings.

  ➡ This also includes the sector of gluon TMD's.

  ➡ Once complete, this will provide a comprehensive profile of what complex spin-orbit structure can look like within QCD.

- Apply the same techniques to the "GTMD's" - the "Mother Functions" which generate both the TMD's and the GPD's

  ➡ The same spin-orbit couplings likely result in specific mixings in both the TMD and GPD sectors.

Summary

- The TMD quark distributions give additional insight into hadronic structure, but they are also sensitive to the gluon fields.
- The high-density limit greatly simplifies the interaction with those gluon fields, bringing them into the perturbative regime.
- The TMD structure of a heavy nucleus factorizes into the nuclear wave function, the nucleonic TMD's, and the calculable gauge factor.
- Spin-orbit coupling in the nucleus results in generic mixing of the TMD's, with the same coupling responsible for multiple mixings.
- This opens new doors to access spin-orbit structure in hadronic systems, both theoretically and experimentally.